

# The Distributed Ontology, Modeling and Specification Language (DOL)

## Language overview

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# Resources

# Resources

- <http://ontoiop.org> Initial standardization initiative
- <https://github.com/tillmo/DOL> repository for development of the DOL standard
- <http://www.omg.org/spec/DOL> future place for DOL standard
- [http://www.omg.org/techprocess/meetings/schedule/OntoIOP\\_RFP.html](http://www.omg.org/techprocess/meetings/schedule/OntoIOP_RFP.html) process at OMG (for members only)
- <http://hets.eu> Tool Hets
- <http://ontohub.org> Ontohub platform

# Motivation

# The Big Picture of Interoperability

Modeling	Specification	Knowledge engineering
Objects/data	Software	Concepts/data
Models	Specifications	Ontologies
Metamodels	Specification languages	Ontology languages

Diversity and the need for interoperability occur at all these levels!

# Ontologies

**Class:** Person

**Class:** Female

**Class:** Woman     **EquivalentTo:** Person **and** Female

**Class:** Man     **EquivalentTo:** Person **and** not Woman

**ObjectProperty:** hasParent

**ObjectProperty:** hasChild InverseOf: hasParent

**ObjectProperty:** hasHusband

**Class:** Mother

**EquivalentTo:** Woman **and** hasChild some Person

**Class:** Father

**EquivalentTo:** Man **and** hasChild some Person

**Class:** Parent

**EquivalentTo:** Father **or** Mother

# Relation between OWL and FOL ontologies

Common practice: annotate OWL ontologies with informal FOL:

- Keet's mereotopological ontology [1],
- Dolce Lite and its relation to full Dolce [2],
- BFO-OWL and its relation to full BFO.

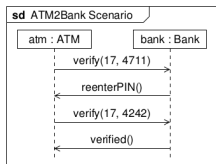
OWL gives better tool support, FOL greater expressiveness.

But: informal FOL axioms are not available for machine processing!

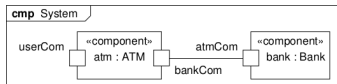
[1] C.M. Keet, F.C. Fernández-Reyes, and A. Morales-González. Representing mereotopological relations in owl ontologies with ontoparts. In *Proceedings of the 9th Extended Semantic Web Conference (ESWC'12), 29-31 May 2012, Heraklion, Crete, Greece*, volume 7295 of *Lecture Notes in Computer Science*, pages 240–254. Springer, 2012.

[2] C. Masolo, S. Borgo, A. Gangemi, N. Guarino, and A. Oltramari. Descriptive ontology for linguistic and cognitive engineering. <http://www.loa.istc.cnr.it/DOLCE.html>.

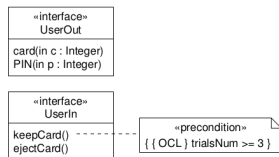
# Models: Is a family of UML models consistent?



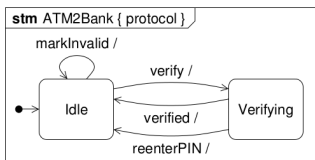
(a) Interaction



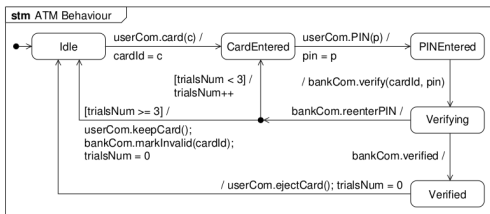
(b) Composite structure



(c) Interfaces

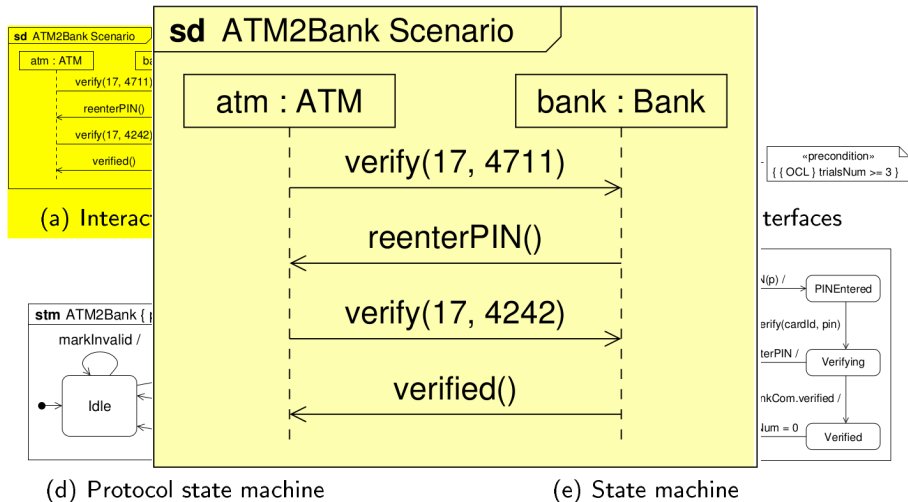


(d) Protocol state machine

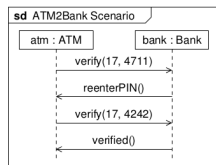


(e) State machine

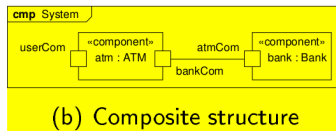
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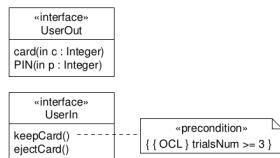
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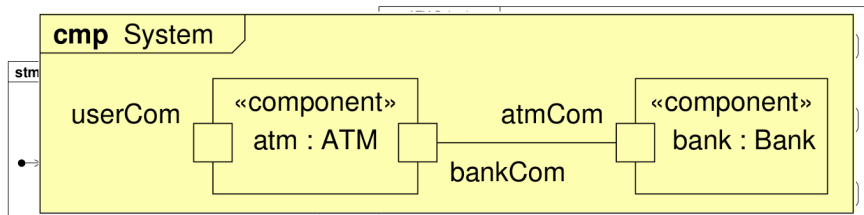
(a) Interaction



(b) Composite structure



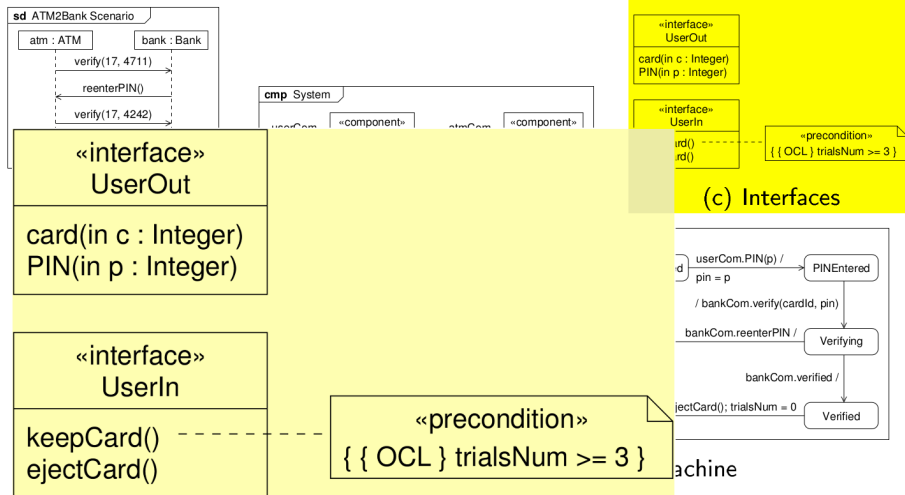
(c) Interfaces



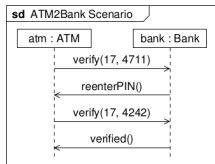
(d) Protocol state machine

(e) State machine

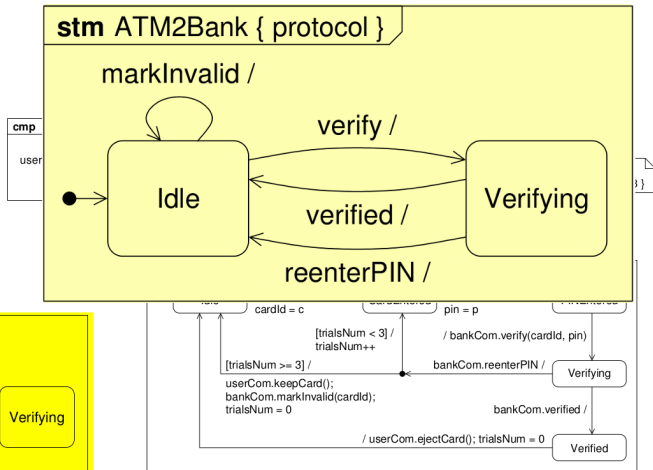
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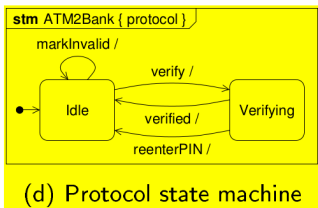
## Models: Is a family of UML models consistent?



(a) Interaction



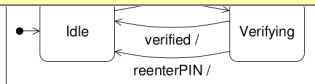
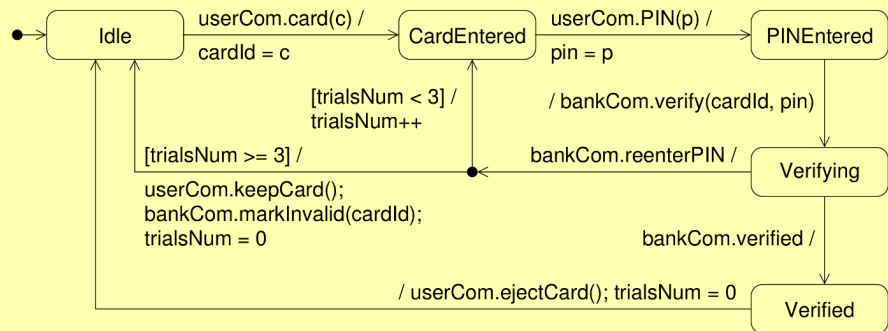
(e) State machine



(d) Protocol state machine

# Models: Is a family of UML models consistent?

## stm ATM Behaviour



(d) Protocol state machine



(e) State machine

# Specification of sorting in CASL/FOL

```

sort Elem
free type List[Elem] ::= [] | __::__(Elem; List[Elem])
pred __<=__ : Elem * Elem
pred __elem__ : Elem * List[Elem]
preds is_ordered : List[Elem];
      permutation : List[Elem] * List[Elem]
op sorter : List[Elem]->List[Elem]
forall x,y:Elem; L,L1,L2:List[Elem]
. not x elem []
. x elem (y :: L) <=> x=y /\ x elem L
. is_ordered([]) . is_ordered(x::[])
. is_ordered(x::y::L) <=> x<=y /\ is_ordered(y::L)
. permutation(L1,L2) <=>
      (forall x:Elem . x elem L1 <=> x elem L2)
. is_ordered(sorter(L))
. permutation(L,sorter(L))

```

# Specification of insert sort in CASL/FOL

```

sort Elem
free type List[Elem] ::= [] | __::__(Elem; List[Elem])
ops insert : Elem*List[Elem] -> List[Elem];
      insert_sort : List[Elem]->List[Elem]
forall x,y:Elem; L:List[Elem]
. insert(x,[]) = x::[]
. x<=y => insert(x,y::L) = x::insert(y,L)
. not x<=y => insert(x,y::L) = y::insert(x,L)
. insert_sort([]) = []
. insert_sort(x::L) = insert(x,insert_sort(L))
  
```

Is insert sort correct w.r.t. the sorting specification?

# What have ontologies, models and specifications in common?

- formalised in some logical system
- signature with non-logical symbols (domain vocabulary)
- axioms expressing the domain-specific facts
- semantics: class of structures (models) interpreting signature symbols in some semantic domain
- we are interested in those structures (models) satisfying the axioms

We henceforth call them “OMS”!

# Motivation: Diversity of Operations on and Relations among OMS

Various operations and relations on OMS are in use:

- **structuring**: union, translation, hiding, ...
- **refinement**
- matching and **alignment**
  - of many OMS covering one domain
- module extraction
  - get **relevant information** out of large OMS
- approximation
  - model in an **expressive** language, **reason fast** in a lightweight one
- ontology-based **database** access/data management
- distributed OMS
  - **bridges** between different modellings

# OntolOp

# Need for a Unifying Meta Language

Not yet another OMS language, but a meta language covering

- diversity of OMS languages
- translations between these
- diversity of operations on and relations among OMS

Current standards like the OWL API or the alignment API only cover parts of this

The  
Ontology, Modeling and Specification  
Integration and Interoperability (OntoOp)  
initiative addresses this

# The OntoOp initiative (ontoiop.org)

- started in 2011 as ISO 17347 within ISO/TC 37/SC 3
- now continued as OMG standard
  - OMG has more experience with **formal semantics**
  - OMG documents will be **freely available**
  - focus extended from ontologies only to **formal models** and **specifications** (i.e. logical theories)
  - request for proposals (RFP) has been issued in December 2013
  - proposals answering RFP due in **December 2014**
- 50 experts participate,  $\sim 15$  have contributed
- OntoOp is open for your ideas, so **join us!**
- Distributed Ontology, Modeling and Specification Language
  - DOL = one specific answer to the RFP requirements
  - there may be other answers to the RFP
  - DOL is based on some **graph of institutions and (co)morphisms**
  - DOL has a **model-level** and a **theory-level semantics**

# DOL

# Overview of DOL

## 1 OMS

- basic OMS (flattenable)
- references to named OMS
- extensions, unions, translations (flattenable)
- reductions, minimization, maximization (elusive)
- approximations, module extractions (flattenable)
- combinations (flattenable)

only OMS with flattenable components are flattenable

## 2 OMS mappings (between OMS)

- interpretations, refinements, alignments, ...

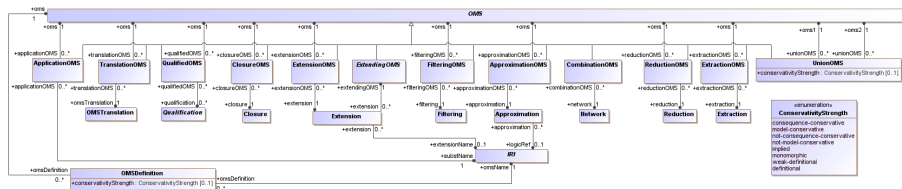
## 3 OMS networks (based on OMS and mappings)

## 4 OMS libraries (based on OMS, mappings, networks)

- OMS definitions (giving a name to an OMS)
- definitions of interpretations, refinements, alignments
- definitions of module relations

# OMS

# Abstract syntax of OMS



# Concrete syntax of OMS

```

BasicOMS                ::= <language and serialization specific>
ClosableOMS             ::= BasicOMS | OMSRef [ImportName]
ImportName              ::= '%(' IRI ')%'
OMSRef                  ::= IRI
ExtendingOMS            ::= ClosableOMS | RelativeClosureOMS
RelativeClosureOMS      ::= ClosureType '{' ClosableOMS '}'
OMS                    ::= ExtendingOMS
                        | OMS Closure
                        | OMS OMSTranslation
                        | OMS Reduction
                        | OMS Approximation
                        | OMS Filtering
                        | OMS 'and' [ConservativityStrength] OMS
                        | OMS 'then' ExtensionOMS
                        | Qualification* ':' GroupOMS
                        | 'combine' NetworkElements [ExcludeExtensions]
                        | GroupOMS
GroupOMS                ::= '{' OMS '}' | OMSRef
  
```

# Basic OMS

- written in **some OMS language** from the logic graph
- semantics is **inherited** from the OMS language
- e.g. in OWL:

**Class:** Woman    **EquivalentTo:** Person **and** Female  
**ObjectProperty:** hasParent

- e.g. in Common Logic:

```
(cl-text PreOrder
  (forall (x) (le x x))
  (forall (x y z)
    (if (and (le x y)
              (le y z))
        (le x z))))
```

# Syntax of extensions

```
BasicOMS ::= <language and serialization specific>
OMS      ::= ...
           | OMS 'then' BasicOMS
           | ...
```

# Extensions

- $O_1$  **then**  $O_2$ : extension of  $O_1$  by new symbols and axioms  $O_2$
- example in OWL:

```
Class Person
Class Female
then
Class: Woman EquivalentTo: Person and Female
```

# Full syntax of extensions

```

BasicOMS          ::= <language and serialization specific>
ClosableOMS       ::= BasicOMS | OMSRef [ImportName]
ExtendingOMS      ::= ClosableOMS | RelativeClosureOMS
OMS               ::= ...
                  | OMS 'then' ExtensionOMS
                  | ...
ExtensionOMS       ::= [ExtConservativityStrength] [ExtensionName] ExtendingOMS
ExtensionName      ::= '%(' IRI ')%'
ExtConservativityStrength ::= '%ccons' | '%mcons'
                  | '%notccons' | '%notmcons'
                  | '%mono' | '%wdef' | '%def'
                  | '%implied'

```

# Extensions with annotations

- $O_1$  **then %mcons**  $O_2$ : model-conservative extension
  - each  $O_1$ -model has an expansion to  $O_1$  **then**  $O_2$
- $O_1$  **then %ccons**  $O_2$ : consequence-conservative extension
  - $O_1$  **then**  $O_2 \models \varphi$  implies  $O_1 \models \varphi$ , for  $\varphi$  in the language of  $O_1$
- $O_1$  **then %def**  $O_2$ : definitional extension
  - each  $O_1$ -model has a **unique** expansion to  $O_1$  **then**  $O_2$
- $O_1$  **then %implies**  $O_2$ : like %mcons, but  $O_2$  must not extend the signature
- example in OWL:

```

Class Person
Class Female
then %def
Class: Woman EquivalentTo: Person and Female
  
```

# References to Named OMS

- **Reference** to an OMS existing on the Web
- written directly as a **URL** (or IRI)
- **Prefixing** may be used for abbreviation

`http://owl.cs.manchester.ac.uk/co-ode-files/  
ontologies/pizza.owl`

`co-ode:pizza.owl`

# Syntax of unions

```
OMS ::= ...  
      | OMS 'and' [ConservativityStrength] OMS  
      | ...
```

# Unions

- $O_1$  **and**  $O_2$ : union of two stand-alone OMS  
(for extensions  $O_2$  needs to be basic)
- Signatures (and axioms) are **united**
- model classes are **intersected**

`algebra:Monoid` **and** `algebra:Commutative`

# Syntax of translations

```

OMS ::= ...
      | OMS OMSTranslation
      | ...
OMSTranslation ::= 'with' SymbolMap
SymbolMap ::= SymbolMapItem ( ',' SymbolMapItem )*
SymbolMapItem ::= Symbol '|->' Symbol
Symbol ::= IRI

```

# Translations

- $O$  **with**  $\sigma$ , where  $\sigma$  is a symbol map (signature morphism)

BankOntology **with** Bank  $\rightarrow$  FinancialBank  
**and**

RiverOntology **with** Bank  $\rightarrow$  RiverBank  
*%% necessary disambiguation when uniting ontologies*

# Full syntax of translations

```

OMS                ::= ...
                   | OMS OMSTranslation
                   | ...
OMSTranslation     ::= 'with' LanguageTranslation* SymbolMap
LanguageTranslation ::= 'translation' OMSLanguageTranslation
SymbolMap          ::= GeneralSymbolMapItem ( ',' GeneralSymbolMapItem )*
GeneralSymbolMapItem ::= Symbol | SymbolMapItem
SymbolMapItem      ::= Symbol '|->' Symbol
Symbol             ::= IRI
LanguageTranslation ::= 'translation' OMSLanguageTranslation
OMSLanguageTranslation ::= OMSLanguageTranslationRef | '|->' LoLaRef
OMSLanguageTranslationRef ::= IRI
LoLaRef             ::= LanguageRef | LogicRef
LanguageRef         ::= IRI
LogicRef            ::= IRI

```

# Translations

- **$O$  with  $\sigma$** , where  $\sigma$  is a signature morphism
- **$O$  with translation  $\rho$** , where  $\rho$  is a **logic translation**

**ObjectProperty:** isProperPartOf

**Characteristics:** Asymmetric

**SubPropertyOf:** isPartOf

**with translation** trans:SR0IQtoCL

**then**

```
(if (and (isProperPartOf x y) (isProperPartOf y z))
      (isProperPartOf x z))
```

```
%% transitivity; can't be expressed in OWL together
%% with asymmetry
```

# Hide – Extract – Forget – Select

	hide/reveal	remove/extract	forget/keep	select/reject
semantic background	model reduct	conservative extension	uniform interpolation	theory filtering
relation to original	interpretable	subtheory	interpretable	subtheory
approach	model level	theory level	theory level	theory level
type of OMS	elusive	flattenable	flattenable	flattenable
signature of result	$= \Sigma$	$\geq \Sigma$	$= \Sigma$	$\geq \Sigma$
change of logic	possible	not possible	possible	not possible
application	specification	ontologies	ontologies	blending

# Syntax of reduction

```

OMS
    ::= ...
    | OMS Reduction
    | ...
Reduction
    ::= 'hide' LogicReduction* SymbolList
    | 'reveal' SymbolList
LogicReduction
    ::= 'along' OMSLanguageTranslation
SymbolList
    ::= Symbol ( ',' Symbol )*
```

# Reduction: Hide/reveal

- intuition: some logical or non-logical symbols are hidden, but the semantic effect of sentences (also those involving these symbols) is kept
- **$O$  reveal  $\Sigma$** , where  $\Sigma$  is a subsignature of that of  $O$
- **$O$  hide  $\Sigma$** , where  $\Sigma$  is a subsignature of that of  $O$
- **$O$  hide along  $\mu$** , where  $\mu$  is a **logic projection**

# Reduction: example

**sort** Elem

**ops** 0:Elem; \_\_+\_\_:Elem\*Elem->Elem; **inv:Elem->Elem**

**forall** x,y,z:elem .  $x+0=x$

.  $x+(y+z) = (x+y)+z$

.  $x+\text{inv}(x)=0$

**hide inv**

Semantics: class of all monoids that can be extended with an inverse, i.e. class of all groups. The effect is second-order quantification:

**sort** Elem

**ops** 0:Elem; \_\_+\_\_:Elem\*Elem->Elem;

**exists inv:Elem->Elem** .

**forall** x,y,z:elem .  $x+0=x$

$\wedge x+(y+z) = (x+y)+z$

$\wedge x+\text{inv}(x)=0$

# Syntax of module extraction

```
OMS ::= ...
      | OMS Extraction
      | ...
Extraction ::= 'extract' InterfaceSignature
            | 'remove' InterfaceSignature
InterfaceSignature ::= SymbolList
SymbolList ::= Symbol ( ',' Symbol )*
```

# Module Extraction: remove/extract

## $O \text{ extract } \Sigma$

- $\Sigma$ : interface signature (subsignature of that of  $O$ )
- $O \text{ extract } \Sigma$  is the minimal depleting  $\Sigma$ -module of  $O$
- Note:  $O$  is a  $\Sigma$ -conservative extension of  $O \text{ extract } \Sigma$ .
- Dually:  $O \text{ remove } \Sigma$  (here,  $\Sigma$  specifies the symbols that are **not** in the interface signature)

# Module Extraction: example

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem; inv:Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . x+inv(x) = 0

remove inv

```

The semantics is the following theory:

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem; inv:Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . x+inv(x) = 0

```

The module needs to be enlarged to the whole OMS.

# Module Extraction: 2nd example

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem; inv:Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . x+inv(x) = 0
                        . exists y:Elem . x+y=0

remove inv

```

The semantics is the following theory:

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . exists y:Elem . x+y=0

```

Here, adding **inv** is conservative.

# Syntax of approximation

```
OMS                ::= ...
                   | OMS Approximation
                   | ...
Approximation      ::= 'forget' InterfaceSignature ['keep' LogicRef]
                   | 'keep' InterfaceSignature ['keep' LogicRef]
                   | 'keep' LogicRef
InterfaceSignature ::= SymbolItems
LogicRef           ::= IRI
```

# Approximation: forget/keep

- $O \text{ keep } \Sigma$ , where  $\Sigma$  is a subsignature of that of  $O$
- $O \text{ keep } \Sigma \text{ keep } L$ , where  $\Sigma$  is a subsignature of that of  $O$ , and  $L$  is a sublogic of that of  $O$
- $O \text{ keep } L$ , where  $L$  is a sublogic of that of  $O$ 
  - intuition: theory of  $O$  is interpolated in smaller signature/logic
- dually
  - $O \text{ forget } \Sigma$
  - $O \text{ forget } \Sigma \text{ keep } L$

# Interpolation: example

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem; inv:Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . x+inv(x) = 0

forget inv
  
```

The semantics is the following theory:

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . exists y:Elem . x+y=0
  
```

Computing interpolants can be hard, even undecidable.

# Syntax of filtering

```
OMS ::= ...
      | OMS Filtering
      | ...
Filtering ::= 'select' SymbolList
            | 'select' BasicOMS
            | 'reject' SymbolList
            | 'reject' BasicOMS
```

# Filtering

- **$O$  select  $T$** , where  $T$  is a subtheory (fragment) of that of  $O$ 
  - intuition: axioms involving only symbols in  $Sig(T)$  are kept
  - moreover, all axioms contained in  $T$  are kept as well
- **$O$  reject  $T$** , where  $T$  is a subtheory (fragment) of that of  $O$ 
  - intuition: all axioms involving symbols in  $Sig(T)$  are deleted
  - moreover, all axioms contained in  $T$  are deleted as well

# Filtering: example

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem; inv:Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
                        . x+inv(x) = 0
reject inv
  
```

The semantics is the following theory:

```

sort Elem
ops 0:Elem; __+__:Elem*Elem->Elem
forall x,y,z:elem . x+0=x
                        . x+(y+z) = (x+y)+z
  
```

# Hide – Extract – Forget – Select

	hide/reveal	remove/extract	forget/keep	select/reject
semantic background	model reduct	conservative extension	uniform interpolation	theory filtering
relation to original	interpretable	subtheory	interpretable	subtheory
approach	model level	theory level	theory level	theory level
type of OMS	elusive	flattenable	flattenable	flattenable
signature of result	$= \Sigma$	$\geq \Sigma$	$= \Sigma$	$\geq \Sigma$
change of logic	possible	not possible	possible	not possible
application	specification	ontologies	ontologies	blending

# Reduction: specification example

```

spec List =  sort Elem
  free type List[Elem] ::= [] | __::__(Elem; List[Elem])
  pred __elem__ : Elem * List[Elem]
  forall x,y:Elem; L,L1,L2:List[Elem]
    . not x elem [] . x elem (y :: L) <=>  x=y /\ x elem L
spec Sorting = List then
  preds is_ordered : List[Elem];
    permutation : List[Elem] * List[Elem]
  op sorter : List[Elem]->List[Elem]
  forall x,y:Elem; L,L1,L2:List[Elem]
    . is_ordered([]) . is_ordered(x::[])
    . is_ordered(x::y::L) <=> x<=y /\ is_ordered(y::L)
    . permutation(L1,L2) <=>
      (forall x:Elem . x elem L1 <=> x elem L2)
    . is_ordered(sorter(L)) . permutation(L,sorter(L))
hide permutation, is_ordered

```

# Relations among the different notions

$$\begin{aligned}
 & \text{Mod}(O \text{ hide } \Sigma) \\
 = & \text{Mod}(O \text{ remove } \Sigma) \upharpoonright_{\text{sig}(O) \setminus \Sigma} \\
 \subseteq & \text{Mod}(O \text{ forget } \Sigma) \\
 \subseteq & \text{Mod}(O \text{ reject } \Sigma)
 \end{aligned}$$

# Pros and Cons

	hide/reveal	remove/extract	forget/keep	select/reject
information loss	none	none	minimal	large
computability	bad	good/depends	depends	easy
signature of result	$= \Sigma$	$\geq \Sigma$	$= \Sigma$	$= \Sigma$
change of logic	possible	not possible	possible	not possible
conceptual simplicity	simple (but unintuitive)	complex	fairly simple	simple

# Syntax of closure

```

ClosableOMS      ::= BasicOMS | OMSRef [ImportName]
ExtendingOMS     ::= ClosableOMS | RelativeClosureOMS
RelativeClosureOMS ::= ClosureType '{' ClosableOMS '}'
OMS              ::= ...
                  | OMS Closure
                  | ...
Closure          ::= ClosureType CircClosure [CircVars]
ClosureType      ::= 'minimize'
                  | 'closed-world'
                  | 'maximize'
                  | 'free'
                  | 'cofree'
CircClosure      ::= Symbol+
CircVars         ::= 'vars' Symbol+

```

# Minimizations (circumscription)

- $O_1$  then minimize  $\{ O_2 \}$
- forces minimal interpretation of non-logical symbols in  $O_2$

**Class:** Block

**Individual:** B1 **Types:** Block

**Individual:** B2 **Types:** Block **DifferentFrom:** B1

**then minimize** {

**Class:** Abnormal

**Individual:** B1 **Types:** Abnormal }

**then**

**Class:** Ontable

**Class:** BlockNotAbnormal **EquivalentTo:**

Block **and not** Abnormal **SubClassOf:** Ontable

**then** %implied

**Individual:** B2 **Types:** Ontable

# Maximizations

- $O_1$  then maximize  $\{ O_2 \}$
- forces maximal interpretation of non-logical symbols in  $O_2$

**Class:** Block

**Individual:** B1 **Types:** Block

**Individual:** B2 **Types:** Block **DifferentFrom:** B1

**then maximize** {

**Class:** Normal

**Individual:** B2 **Types:** Normal }

**then**

**Class:** Ontable **SubClassOf:** Block **and** Normal

**then %implied**

**Individual:** B1 **Types:** not Ontable

# Freeness

- $O_1$  **then free**  $\{ O_2 \}$
- forces initial interpretation of non-logical symbols in  $O_2$

```

sort Elem
then free {
  sort Bag
  ops mt:Bag;
    __union__:Bag*Bag->Bag, assoc, comm, unit mt
  }

```

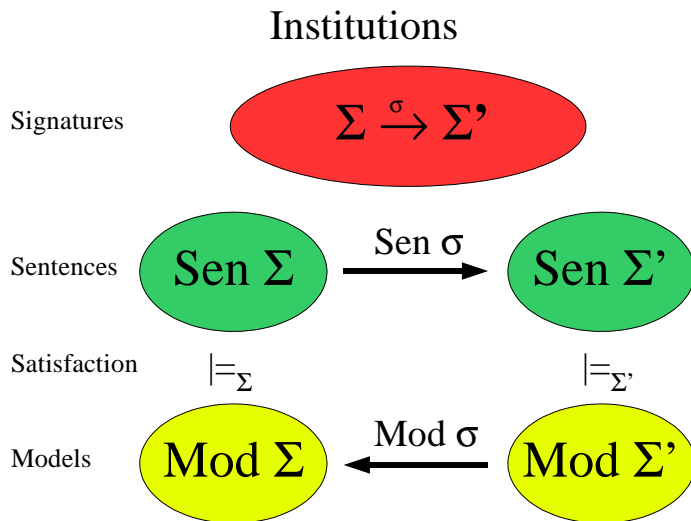
# Cofreeness

- $O_1$  **then cofree**  $\{ O_2 \}$
- forces final interpretation of non-logical symbols in  $O_2$

```
sort Elem
then cofree {
  sort Stream
  ops head:Stream->Elem;
      tail:Stream->Stream
}
```

# Semantics of OMS

# Institutions (intuition)



# Institutions (formal definition)

An **institution**  $\mathcal{I} = \langle \mathbf{Sign}, \mathbf{Sen}, \mathbf{Mod}, \langle \models_{\Sigma} \rangle_{\Sigma \in |\mathbf{Sign}|} \rangle$  consists of:

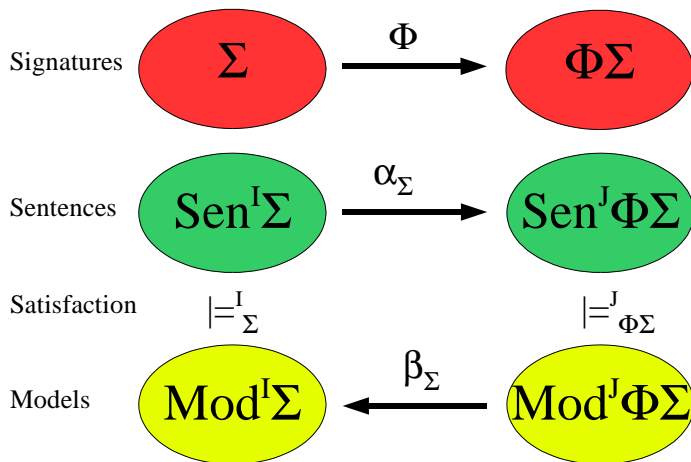
- a category **Sign** of **signatures**;
- a functor **Sen**: **Sign**  $\rightarrow$  **Set**, giving a set **Sen**( $\Sigma$ ) of  **$\Sigma$ -sentences** for each signature  $\Sigma \in |\mathbf{Sign}|$ , and a function **Sen**( $\sigma$ ): **Sen**( $\Sigma$ )  $\rightarrow$  **Sen**( $\Sigma'$ ) that yields  **$\sigma$ -translation** of  $\Sigma$ -sentences to  $\Sigma'$ -sentences for each  $\sigma: \Sigma \rightarrow \Sigma'$ ;
- a functor **Mod**: **Sign**<sup>op</sup>  $\rightarrow$  **Set**, giving a set **Mod**( $\Sigma$ ) of  **$\Sigma$ -models** for each signature  $\Sigma \in |\mathbf{Sign}|$ , and a functor  $-\downarrow_{\sigma} = \mathbf{Mod}(\sigma): \mathbf{Mod}(\Sigma') \rightarrow \mathbf{Mod}(\Sigma)$ ; for each  $\sigma: \Sigma \rightarrow \Sigma'$ ;
- for each  $\Sigma \in |\mathbf{Sign}|$ , a **satisfaction relation**  
 $\models_{\mathcal{I}, \Sigma} \subseteq \mathbf{Mod}(\Sigma) \times \mathbf{Sen}(\Sigma)$

such that for any signature morphism  $\sigma: \Sigma \rightarrow \Sigma'$ ,  $\Sigma$ -sentence  $\varphi \in \mathbf{Sen}(\Sigma)$  and  $\Sigma'$ -model  $M' \in \mathbf{Mod}(\Sigma')$ :

$$M' \models_{\mathcal{I}, \Sigma'} \sigma(\varphi) \iff M'|_{\sigma} \models_{\mathcal{I}, \Sigma} \varphi \quad [\text{Satisfaction condition}]$$

# Institution comorphisms (embeddings, encodings)

## Institution comorphisms



# Institution comorphisms (embeddings, encodings)

## Definition

Let  $\mathcal{I} = \langle \mathbf{Sign}, \mathbf{Sen}, \mathbf{Mod}, \langle \models_{\Sigma} \rangle_{\Sigma \in |\mathbf{Sign}|} \rangle$  and  $\mathcal{I}' = \langle \mathbf{Sign}', \mathbf{Sen}', \mathbf{Mod}', \langle \models'_{\Sigma'} \rangle_{\Sigma' \in |\mathbf{Sign}'|} \rangle$  be institutions. An *institution comorphism*  $\rho: \mathcal{I} \rightarrow \mathcal{I}'$  consists of:

- a functor  $\Phi: \mathbf{Sign} \rightarrow \mathbf{Sign}'$ ;
- a natural transformation  $\alpha: \mathbf{Sen} \rightarrow \Phi; \mathbf{Sen}'$ , and
- a natural transformation  $\beta: (\Phi)^{op}; \mathbf{Mod}' \rightarrow \mathbf{Mod}$ ,

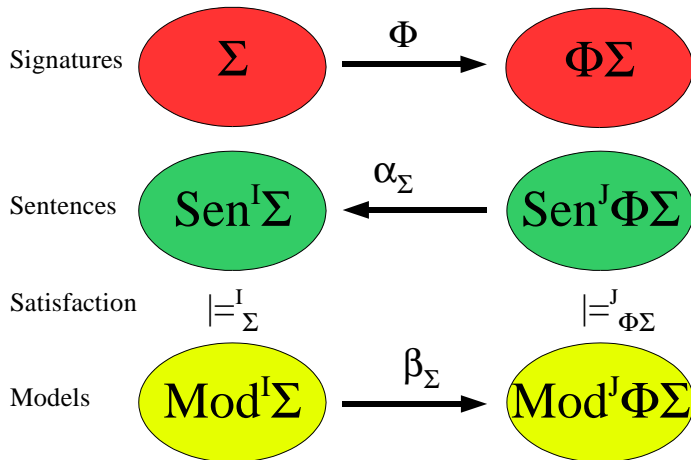
such that for any  $\Sigma \in |\mathbf{Sign}|$ , any  $\varphi \in \mathbf{Sen}(\Sigma)$  and any  $M' \in \mathbf{Mod}'(\Phi(\Sigma))$ :

$$M' \models'_{\Phi(\Sigma)} \alpha_{\Sigma}(\varphi) \iff \beta_{\Sigma}(M') \models_{\Sigma} \varphi$$

[Satisfaction condition]

# Institution morphisms (projections)

## Institution morphisms



# Institution morphisms (projections)

## Definition

Let  $\mathcal{I} = \langle \mathbf{Sign}, \mathbf{Sen}, \mathbf{Mod}, \langle \models_{\Sigma} \rangle_{\Sigma \in |\mathbf{Sign}|} \rangle$  and  $\mathcal{I}' = \langle \mathbf{Sign}', \mathbf{Sen}', \mathbf{Mod}', \langle \models'_{\Sigma'} \rangle_{\Sigma' \in |\mathbf{Sign}'|} \rangle$  be institutions. An *institution morphism*  $\mu: \mathcal{I} \rightarrow \mathcal{I}'$  consists of:

- a functor  $\mu^{Sign}: \mathbf{Sign} \rightarrow \mathbf{Sign}'$ ;
- a natural transformation  $\mu^{Sen}: \mu^{Sign}; \mathbf{Sen}' \rightarrow \mathbf{Sen}$ , and
- a natural transformation  $\mu^{Mod}: \mathbf{Mod} \rightarrow (\mu^{Sign})^{op}; \mathbf{Mod}'$ ,

such that for any signature  $\Sigma \in |\mathbf{Sign}|$ , any  $\varphi' \in \mathbf{Sen}'(\mu^{Sign}(\Sigma))$  and any  $M \in \mathbf{Mod}(\Sigma)$ :

$$M \models_{\Sigma} \mu_{\Sigma}^{Sen}(\varphi') \iff \mu_{\Sigma}^{Mod}(M) \models'_{\mu^{Sign}(\Sigma)} \varphi'$$

[Satisfaction condition]

# Unions, differences and inclusive institutions

We assume that for each institution, there exists (possibly partial) union and difference operations on signatures. E.g. an inclusion system on signatures would be a good framework where this can be required.

## Definition (adopted from Goguen, Roşu)

An *weakly inclusive category* is a category having a broad subcategory which is a partially ordered class.

An *weakly inclusive institution* is one with an inclusive signature category such that the sentence functor preserves inclusions.

We also assume that model categories are weakly inclusive.

$M|_{\Sigma}$  means  $M|_{\iota}$  where  $\iota : \Sigma \rightarrow \text{Sig}(M)$  is the inclusion.

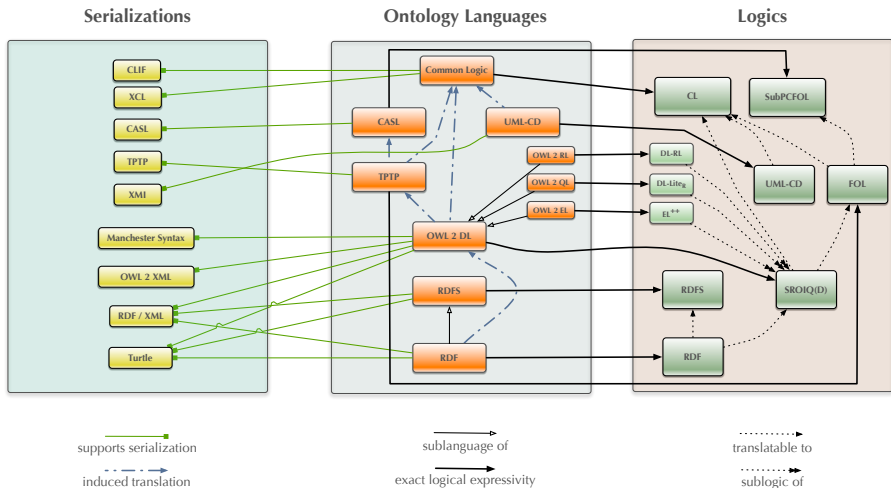
# Heterogeneous logical environments

A heterogeneous logical environment consists of

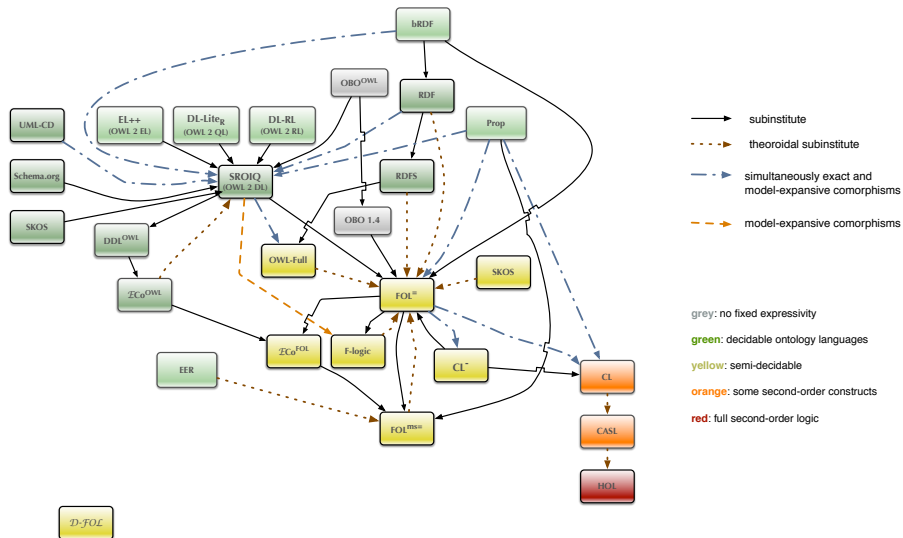
- a logic graph, consisting of institutions, institution comorphisms (translations) and institution morphisms (projections),
- an OMS language graph, and
- supports relations.

The support relations specify which language supports which logics and which serializations, and which language translation supports which logic translation or reduction.

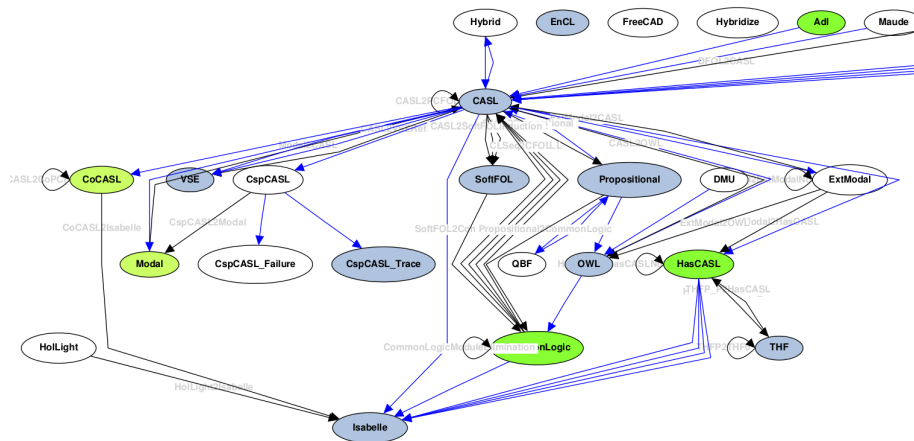
Moreover, each language has a default logic and a default serialization. There are also default translations.



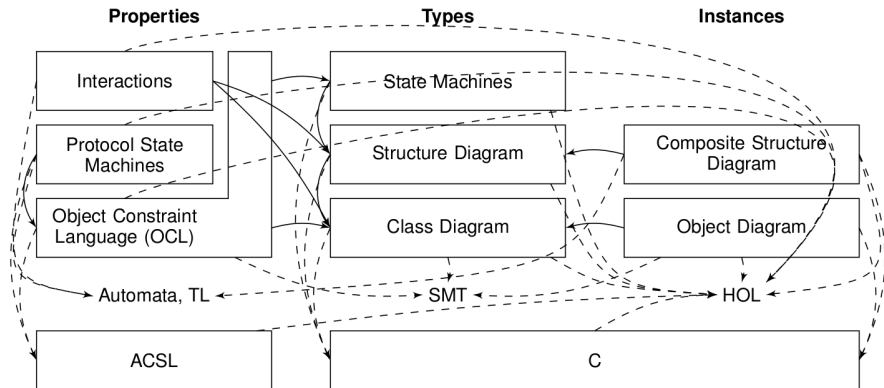
# Ontologies: An Initial Logic Graph



# Specifications: An Initial Logic Graph



# UML models: An Initial Logic Graph



# Semantic domains of DOL

- semantics of a flattenable OMS has form  $(I, \Sigma, \Psi)$  (**theory-level**)
- semantics of an elusive OMS has form  $(I, \Sigma, \mathcal{M})$  (**model-level**)
  - institution  $I$
  - signature  $\Sigma$  in  $I$
  - set  $\Psi$  of  $\Sigma$ -sentences
  - class  $\mathcal{M}$  of  $\Sigma$ -models

We can obtain the model-level semantics from the theory-level semantics by taking  $\mathcal{M} = \{M \in \mathbf{Mod}(\Sigma) \mid M \models \Psi\}$ .

- **semantics of an OMS declaration/relation** has form  $\Gamma: IRI \longrightarrow (OMS \uplus OMS \times OMS \times SigMor)$ 
  - $OMS$  is the class of all triples  $(I, \Sigma, \Psi)$ ,  $(I, \Sigma, \mathcal{M})$
  - for interpretations etc., domain, codomain and signature morphism is recorded:  $OMS \times OMS \times SigMor$

# Semantics of basic OMS

We assume that  $\llbracket O \rrbracket_{basic} = (I, \Sigma, \Psi)$  for some OMS language based on  $I$ . The semantics consists of

- the **institution**  $I$
- a **signature**  $\Sigma$  in  $I$
- a set  $\Psi$  of  $\Sigma$ -**sentences**

This direct leads to a theory-level semantics for the OMS:

$$\llbracket O \rrbracket_{\Gamma}^T = \llbracket O \rrbracket_{basic}$$

Generally, if a **theory-level** semantics is given:  $\llbracket O \rrbracket_{\Gamma}^T = (I, \Sigma, \Psi)$ , this leads to a **model-level semantics** as well:

$$\llbracket O \rrbracket_{\Gamma}^M = (I, \Sigma, \{M \in Mod(\Sigma) \mid M \models \Psi\})$$

# Semantics of extensions

$O_1$  flattenable  $\llbracket O_1 \text{ then } O_2 \rrbracket_{\Gamma}^T = (I, \Sigma_1 \cup \Sigma_2, \Psi_1 \cup \Psi_2)$

where

- $\llbracket O_1 \rrbracket_{\Gamma}^T = (I, \Sigma_1, \Psi_1)$
- $\llbracket O_2 \rrbracket_{\text{basic}} = (I, \Sigma_2, \Psi_2)$

$O_1$  elusive  $\llbracket O_1 \text{ then } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_1 \cup \Sigma_2, \mathcal{M}')$

where

- $\llbracket O_1 \rrbracket_{\Gamma}^M = (I, \Sigma_1, \mathcal{M}_1)$
- $\llbracket O_2 \rrbracket_{\text{basic}} = (I, \Sigma_2, \Psi_2)$
- $\mathcal{M}' = \{M \in \mathbf{Mod}(\Sigma_1 \cup \Sigma_2) \mid M \models \Psi_2, M|_{\Sigma_1} \in \mathcal{M}_1\}$

# Semantics of extensions (cont'd)

`%mcons` (`%def`, `%mono`) leads to the additional requirement that

*each model in  $\mathcal{M}_1$  has a (unique, unique up to isomorphism)  $\Sigma_1 \cup \Sigma_2$ -expansion to a model in  $\mathcal{M}'$ .*

`%implies` leads to the additional requirements that

$\Sigma_2 \subseteq \Sigma_1$  and  $\mathcal{M}' = \mathcal{M}_1$ .

`%ccons` leads to the additional requirement that

$\mathcal{M}' \models \varphi$  implies  $\mathcal{M}_1 \models \varphi$  for any  $\Sigma_1$ -sentence  $\varphi$ .

## Theorem

*`%mcons` implies `%ccons`, but not vice versa.*

# References to Named OMS

- **Reference** to an OMS existing on the Web
- written directly as a **URL** (or IRI)
- **Prefixing** may be used for abbreviation

`http://owl.cs.manchester.ac.uk/co-ode-files/ontologies/pizza.owl`

`co-ode:pizza.owl`

Semantics Reference to Named OMS:  $\llbracket iri \rrbracket_{\Gamma} = \Gamma(iri)$

# Semantics of unions

$O_1, O_2$  flattenable  $\llbracket O_1 \text{ and } O_2 \rrbracket_{\Gamma}^T = (I, \Sigma_1 \cup \Sigma_2, \Psi_1 \cup \Psi_2)$ , where

- $\llbracket O_i \rrbracket_{\Gamma}^T = (I, \Sigma_i, \Psi_i) \ (i = 1, 2)$

one of  $O_1, O_2$  elusive  $\llbracket O_1 \text{ and } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_1 \cup \Sigma_2, \mathcal{M})$ , where

- $\llbracket O_i \rrbracket_{\Gamma}^M = (I, \Sigma_i, \mathcal{M}_i) \ (i = 1, 2)$
- $\mathcal{M} = \{M \in \mathbf{Mod}(\Sigma_1 \cup \Sigma_2) \mid M|_{\Sigma_i} \in \mathcal{M}_i, i = 1, 2\}$

# Semantics of translations

*O* flattenable Let  $\llbracket O \rrbracket_{\Gamma}^T = (I, \Sigma, \Psi)$

- homogeneous translation

$$\llbracket O \text{ with } \sigma : \Sigma \rightarrow \Sigma' \rrbracket_{\Gamma}^T = (I, \Sigma', \sigma(\Psi))$$

- heterogeneous translation

$$\llbracket O \text{ with translation } \rho : I \rightarrow I' \rrbracket_{\Gamma}^T = (I', \rho^{Sig}(\Sigma), \rho^{Sen}(\Psi))$$

*O* elusive Let  $\llbracket O \rrbracket_{\Gamma}^M = (I, \Sigma, \mathcal{M})$

- homogeneous translation

$$\llbracket O \text{ with } \sigma : \Sigma \rightarrow \Sigma' \rrbracket_{\Gamma}^M = (I, \Sigma', \mathcal{M}')$$

where  $\mathcal{M}' = \{M \in \mathbf{Mod}(\Sigma') \mid M|_{\sigma} \in \mathcal{M}\}$

- heterogeneous translation

$$\llbracket O \text{ with translation } \rho : I \rightarrow I' \rrbracket_{\Gamma}^M = (I', \rho^{Sig}(\Sigma), \mathcal{M}') \text{ where}$$

$$\mathcal{M}' = \{M \in \mathbf{Mod}^{I'}(\rho^{Sig}(\Sigma)) \mid \rho^{Mod}(M) \in \mathcal{M}\}$$

# Hide – Extract – Forget – Select

	hide/reveal	remove/extract	forget/keep	select/reject
semantic background	model reduct	conservative extension	uniform interpolation	theory filtering
relation to original	interpretable	subtheory	interpretable	subtheory
approach	model level	theory level	theory level	theory level
type of OMS	elusive	flattenable	flattenable	flattenable
signature of result	$= \Sigma$	$\geq \Sigma$	$= \Sigma$	$\geq \Sigma$
change of logic	possible	not possible	possible	not possible
application	specification	ontologies	ontologies	blending

# Semantics of reductions

Let  $\llbracket O \rrbracket_{\Gamma}^M = (I, \Sigma, \mathcal{M})$

- homogeneous reduction

$$\llbracket O \text{ reveal } \Sigma' \rrbracket_{\Gamma}^M = (I, \Sigma', \mathcal{M}|_{\Sigma'})$$

$$\llbracket O \text{ hide } \Sigma' \rrbracket_{\Gamma}^M = \llbracket O \text{ reveal } \Sigma \setminus \Sigma' \rrbracket_{\Gamma}^M$$

- heterogeneous reduction

$$\llbracket O \text{ hide along } \rho : I \rightarrow I' \rrbracket_{\Gamma}^M = (I', \rho^{Sig}(\Sigma), \rho^{Mod}(\mathcal{M}))$$

$\mathcal{M}|_{\Sigma'}$  may be impossible to capture by a theory (even if  $\mathcal{M}$  is).

The proof calculus for refinements involving reduction needs invention of some OMS  $O''$ :

$$\frac{O \rightsquigarrow O''}{O \text{ hide } \Sigma \rightsquigarrow O'} \quad \text{if } \iota : O' \longrightarrow O'' \text{ is a conservative extension}$$

where  $\iota : \Sigma \rightarrow Sig(O)$  is the inclusion

# Modules

## Definition

$O' \subseteq O$  is a  **$\Sigma$ -module** of (flat)  $O$  iff  $O$  is a model-theoretic  $\Sigma$ -conservative extension of  $O'$ , i.e. for every model  $M$  of  $O'$ ,  $M|_{\Sigma}$  can be expanded to an  $O$ -model.

# Depleting modules

## Definition

Let  $O_1$  and  $O_2$  be two OMS and  $\Sigma \subseteq \text{Sig}(O_i)$ .

Then  $O_1$  and  $O_2$  are  $\Sigma$ -inseparable ( $O_1 \equiv_{\Sigma} O_2$ ) iff

$$\text{Mod}(O_1)|_{\Sigma} = \text{Mod}(O_2)|_{\Sigma}$$

## Definition

$O' \subseteq O$  is a **depleting  $\Sigma$ -module** of (flat)  $O$  iff  $O \setminus O' \equiv_{\Sigma \cup \text{Sig}(O')} \emptyset$ .

## Theorem

- ① *Depleting  $\Sigma$ -modules are  $\Sigma$ -conservative.*
- ② *The minimum depleting  $\Sigma$ -module always exists.*

# Semantics of module extraction (remove/extract)

Note:  $O$  must be flattenable!

Let  $\llbracket O \rrbracket_r^T = (I, \Sigma, \Psi)$ .

$\llbracket O \text{ extract } \Sigma_1 \rrbracket_r^T = (I, \Sigma_2, \Psi_2)$

where  $(\Sigma_2, \Psi_2) \subseteq (\Sigma, \Psi)$  is the minimum depleting  $\Sigma_1$ -module of  $(\Sigma, \Psi)$

$\llbracket O \text{ remove } \Sigma_1 \rrbracket_r^T = \llbracket O \text{ extract } \Sigma \setminus \Sigma_1 \rrbracket_r^T$

Tools can extract any module (i.e. using locality). Any two modules will have the same  $\Sigma$ -consequences.

# Semantics of interpolation (forget/keep)

Note:  $O$  must be flattenable!

Let  $\llbracket O \rrbracket_r^T = (I, \Sigma, \Psi)$ .

- homogeneous interpolation

$$\llbracket O \text{ keep in } \Sigma' \rrbracket_r^T = (I, \Sigma', \{\varphi \in \text{Sen}(\Sigma') \mid \Psi \models \varphi\})$$

(note: any logically equivalent theory will also do)

$$\llbracket O \text{ forget } \Sigma' \rrbracket_r^T = \llbracket O \text{ keep in } \Sigma \setminus \Sigma' \rrbracket_r^T$$

- heterogeneous interpolation

$$\begin{aligned} \llbracket O \text{ keep in } \Sigma' \text{ with } I' \rrbracket_r^T = \\ (I', \Sigma', \{\varphi \in \text{Sen}'(\Sigma') \mid \Psi \models \rho^{\text{Sen}}(\varphi)\}) \end{aligned}$$

where  $\rho : I' \rightarrow I$  is the inclusion

and  $\Sigma'$  is such that  $\rho^{\text{Sig}}(\Sigma') \subseteq \Sigma$

$$\llbracket O \text{ forget } \Sigma' \text{ with } I' \rrbracket_r^T = \llbracket O \text{ keep in } \Sigma \setminus \Sigma' \text{ with } I' \rrbracket_r^T$$

# Semantics of select/reject

Note:  $O$  must be flattenable!

Let  $\llbracket O \rrbracket_r^T = (I, \Sigma, \Psi)$ .

$\llbracket O \text{ select } (\Sigma', \Phi) \rrbracket_r^T = (I, \Sigma, \text{Sen}(\iota)^{-1}(\Psi) \cup \Phi)$

where  $\iota : \Sigma' \rightarrow \Sigma$  is the inclusion

$\llbracket O \text{ reject } (\Sigma', \Phi) \rrbracket_r^T = (I, \Sigma \setminus \Sigma', \text{Sen}(\iota)^{-1}(\Psi) \setminus \Phi)$

where  $\iota : \Sigma \setminus \Sigma' \rightarrow \Sigma$  is the inclusion

# Hide – Extract – Forget – Select

	hide/reveal	remove/extract	forget/keep	select/reject
semantic background	model reduct	conservative extension	uniform interpolation	theory filtering
relation to original	interpretable	subtheory	interpretable	subtheory
approach	model level	theory level	theory level	theory level
type of OMS	elusive	flattenable	flattenable	flattenable
signature of result	$= \Sigma$	$\geq \Sigma$	$= \Sigma$	$\geq \Sigma$
change of logic	possible	not possible	possible	not possible
application	specification	ontologies	ontologies	blending

# Semantics of minimizations

Let  $\llbracket O_1 \rrbracket_{\Gamma}^M = (I, \Sigma_1, \mathcal{M}_1)$

Let  $\llbracket O_1 \text{ then } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_2, \mathcal{M}_2)$

Then

$$\llbracket O_1 \text{ then minimize } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_2, \mathcal{M})$$

where

$$\mathcal{M} = \{M \in \mathcal{M}_2 \mid M \text{ is minimal in } \{M' \in \mathcal{M}_2 \mid M'|_{\Sigma_1} = M|_{\Sigma_1}\}\}$$

Dually: maximization.

# Semantics of freeness

Let  $\llbracket O_1 \rrbracket_{\Gamma}^M = (I, \Sigma_1, \mathcal{M}_1)$

Let  $\llbracket O_1 \text{ then } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_2, \mathcal{M}_2)$

Let  $\iota : \Sigma_1 \rightarrow \Sigma_2$  be the inclusion

Then

$$\llbracket O_1 \text{ then free } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_2, \mathcal{M})$$

where  $\mathcal{M} = \{M \in \mathcal{M}_2 \mid M \text{ is } Mod(\iota)\text{-free over } M|_{\iota} \text{ with unit } id\}$

Given a functor  $G : \mathbf{B} \rightarrow \mathbf{A}$ , an object  $B \in \mathbf{B}$  is called *G-free (with unit  $\eta_A : A \rightarrow G(B)$ ) over  $A \in \mathbf{A}$* , if for any object  $B' \in \mathbf{B}$  and any morphism  $h : A \rightarrow G(B')$ , there is a unique morphism  $h^{\#} : B \rightarrow B'$  such that  $\eta_A; G(h^{\#}) = h$ .

$$\begin{array}{ccc}
 A & \xrightarrow{\eta_A} & G(B) \\
 & \searrow h & \swarrow G(h^{\#}) \\
 & G(B') &
 \end{array}$$

# Semantics of cofreeness

Let  $\llbracket O_1 \rrbracket_{\Gamma}^M = (I, \Sigma_1, \mathcal{M}_1)$

Let  $\llbracket O_1 \text{ then } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_2, \mathcal{M}_2)$

Let  $\iota : \Sigma_1 \rightarrow \Sigma_2$  be the inclusion

Then

$$\llbracket O_1 \text{ then cofree } O_2 \rrbracket_{\Gamma}^M = (I, \Sigma_2, \mathcal{M})$$

$$\mathcal{M} = \{M \in \mathcal{M}_2 \mid M \text{ is } Mod(\iota)\text{-cofree over } M|_{\iota} \text{ with counit } id\}$$

Given a functor  $G : \mathbf{B} \rightarrow \mathbf{A}$ , an object  $B \in \mathbf{B}$  is called *G-cofree* (with counit  $\varepsilon_A : G(B) \rightarrow A$ ) over  $A \in \mathbf{A}$ , if for any object  $B' \in \mathbf{B}$  and any morphism  $h : G(B') \rightarrow A$ , there is a unique morphism  $h^{\#} : B' \rightarrow B$  such that  $G(h^{\#}); \varepsilon_A = h$ .

$$\begin{array}{ccc}
 A & \xleftarrow{\varepsilon_A} & G(B) \\
 & \nwarrow h & \nearrow G(h^{\#}) \\
 & G(B') &
 \end{array}$$

# OMS Libraries

# Syntax of DOL libraries

```

Document          ::= DOLLibrary | NativeDocument
NativeDocument    ::= <language and serialization specific>
DOLLibrary        ::= [PrefixMap] 'library' LibraryName
                   Qualification LibraryItem*
LibraryItem       ::= LibraryImport | Definition | Qualification
Definition        ::= OMSDefinition | NetworkDefinition | MappingDefinition
LibraryImport     ::= 'import' LibraryName
Qualification      ::= LanguageQualification
                   | LogicQualification
                   | SyntaxQualification
LanguageQualification ::= 'language' LanguageRef
LogicQualification  ::= 'logic' LogicRef
SyntaxQualification ::= 'serialization' SyntaxRef
OMSDefinition      ::= OMSkeyword OMSName '='
                   [ConservativityStrength] OMS 'end'
OMSkeyword         ::= 'ontology'
                   | 'onto'
                   | 'specification'
                   | 'spec'
                   | 'model'
                   | 'oms'

```

# OMS definitions

- **OMS** *IRI* = *O* **end**
- assigns name *IRI* to OMS *O*, for later reference  
 $\Gamma(IRI) := \llbracket O \rrbracket_{\Gamma}$

```
ontology co-code:Pizza =
  Class: VegetarianPizza
  Class: VegetableTopping
  ObjectProperty: hasTopping
  . . .
end
```

# Syntax of mappings

```

MappingDefinition ::= InterpretationDefinition
                  | EntailmentDefinition
                  | EquivalenceDefinition
                  | ModuleRelDefinition
                  | AlignmentDefinition
InterpretationDefinition ::= InterpretationKeyword
                           InterpretationName
                           [Conservative] ':'
                           InterpretationType '='
                           LanguageTranslation*
                           [SymbolMap] 'end'
InterpretationKeyword ::= 'interpretation' | 'view' | 'refinement'
InterpretationName ::= IRI
InterpretationType ::= GroupOMS 'to' GroupOMS

```

# Interpretations (refinements)

- **interpretation**  $Id : O_1 \text{ to } O_2 = \sigma$
- $\sigma$  is a signature morphism or a logic translation
- expresses that  $O_2$  logically implies  $\sigma(O_1)$

**interpretation**  $i : \text{TotalOrder to Nat} = \text{Elem} \mapsto \text{Nat}$

**interpretation** geometry\_of\_time %mcons :

*%% Interpretation of linearly ordered time intervals.*

int:owltime\_le

*%% ... that begin and end with an instant as lines*

*%% that are incident with linearly ...*

**to** { ord:linear\_ordering **and** bi:complete\_graphical

*%% ... ordered points in a special geometry, ...*

**and** int:mappings/owltime\_interval\_reduction }

= ProperInterval  $\mapsto$  Interval **end**

# An interpretation in UML

```
%prefix( :      <http://www.example.org/uml#>
           uml:   <http://www.uml.org/spec/UML/>
%% descriptions of logics ...
           log:   <http://www.omg.org/spec/DOL/logics/>

logic log:uml

interpretation abstract_to_concrete_atm :
  psm to
  {  atm with Idle |-> Idle, CardEntered |-> Idle,
      PINEntered |-> Idle, Verified |-> Idle,
      Verifying |-> Verifying
    hide card, PIN  } = translation psm2atm
end
```

# An interpretation in CASL

**spec** InsertSort =

List

**then**

**ops** insert : Elem\*List[Elem] -> List[Elem];

insert\_sort : List[Elem]->List[Elem]

**vars** x,y:Elem; L:List[Elem]

. insert(x,[]) = x::[]

. insert(x,y::L) = x::insert(y,L) when  $x \leq y$  else y::L

. insert\_sort([]) = []

. insert\_sort(x::L) = insert(x,insert\_sort(L))

**hide** insert

**interpretation** InsertSortCorrectness :

Sorting **to** InsertSort =

sorter | -> insert\_sort

# Semantics of interpretations

Let  $\llbracket O_i \rrbracket_{\Gamma}^M = (I, \Sigma_i, \mathcal{M}_i)$  ( $i = 1, 2$ )

$\llbracket \text{interpretation } IRI : O_1 \text{ to } O_2 = \sigma \rrbracket_{\Gamma}^M$

is defined iff

$$\text{Mod}(\sigma)(\mathcal{M}_2) \subseteq \mathcal{M}_1$$

In this case,  $\Gamma(IRI) := ((I, \Sigma_1, \mathcal{M}_1), (I, \Sigma_2, \mathcal{M}_2), \sigma)$ .

# Syntax of OMS networks (diagrams)

```

NetworkDefinition ::= 'network' NetworkName '='
                    [ConservativityStrength] Network
NetworkName       ::= IRI
Network           ::= NetworkElements [ExcludedElements]
NetworkElements   ::= NetworkElement ( ',' NetworkElement )*
NetworkElement    ::= [Id ':' ] ElementRef
ExcludedElements  ::= 'excluding' ExcludedElement ( ',' ExcludedElement )*
ExcludedElement   ::= PathReference | ElementRef
PathReference     ::= IRI '->' IRI
ElementRef        ::= IRI
Id                ::= Letter Letter0rDigit*

```

# OMS networks (diagrams)

**network**  $N =$

$N_1, \dots, N_m, O_1, \dots, O_n, M_1, \dots, M_p$

**excluding**  $N'_1, \dots, N'_i, O'_1, \dots, O'_j, M'_1, \dots, M'_k$

- $N_i$  are other networks
- $O_i$  are OMS (possibly prefixed with labels, like  $n : O$ )
- $M_i$  are mappings (views, interpretations)

# Combinations

- **combine**  $N$
- $N$  is a network
- semantics is the (a) **colimit** of the diagram  $N$

**ontology** AlignedOntology1 =  
**combine**  $N$

There is a natural semantics of diagrams: compatible families of models.

Then in exact institutions, models of diagrams are in bijective correspondence to models of the colimit.

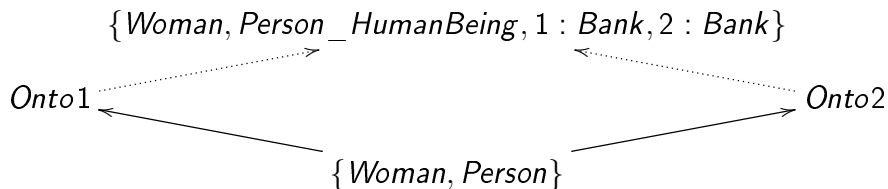
# Sample combination

```

ontology Source =
  Class: Person
  Class: Woman SubClassOf: Person
ontology Onto1 =
  Class: Person           Class: Bank
  Class: Woman SubClassOf: Person
interpretation I1 : Source to Onto1 =
  Person |-> Person, Woman |-> Woman
ontology Onto2 =
  Class: HumanBeing      Class: Bank
  Class: Woman SubClassOf: HumanBeing
interpretation I2 : Source to Onto2 =
  Person |-> HumanBeing, Woman |-> Woman
ontology CombinedOntology =
  combine Source, Onto1, Onto2, I1, I2

```

# Resulting colimit



# Syntax of alignments

```

AlignmentDefinition ::= 'alignment' AlignmentName
                    [AlignmentCardinalityPair] ':'
                    AlignmentType
                    ['=' Correspondence ( ',' Correspondence )*]
                    ['assuming' AlignmentSemantics] 'end'

AlignmentName       ::= IRI
AlignmentCardinalityPair ::= AlignmentCardinalityForward
                             AlignmentCardinalityBackward
AlignmentCardinalityForward ::= AlignmentCardinality
AlignmentCardinalityBackward ::= AlignmentCardinality
AlignmentCardinality ::= '1' | '?' | '+' | '*'
AlignmentType        ::= GroupOMS 'to' GroupOMS
AlignmentSemantics   ::= 'SingleDomain'
                       | 'GlobalDomain'
                       | 'ContextualizedDomain'
Correspondence       ::= CorrespondenceBlock | SingleCorrespondence | '*'
CorrespondenceBlock  ::= 'relation' [Relation] [Confidence] '{'
                       Correspondence ( ',' Correspondence )* '}'
SingleCorrespondence ::= SymbolRef [Relation] [Confidence] SymbolRef
GeneralizedTerm      ::= SymbolRef
Relation              ::= '>' | '<' | '=' | '%' | 'ni' | 'in' | IRI
Confidence            ::= Double

```

# Alignments

- **alignment** *Id card<sub>1</sub> card<sub>2</sub> : O<sub>1</sub> to O<sub>2</sub> = c<sub>1</sub>, ... c<sub>n</sub>*  
 assuming SingleDomain | GlobalDomain |  
 ContextualizedDomain
- *card<sub>i</sub>* is (optionally) one of 1, ?, +, \*
- the *c<sub>i</sub>* are correspondences of form *sym<sub>1</sub> rel conf sym<sub>2</sub>*
  - *sym<sub>i</sub>* is a symbol from *O<sub>i</sub>*
  - *rel* is one of >, <, =, %, ∃, ∈, ↦, or an *Id*
  - *conf* is an (optional) confidence value between 0 and 1

Syntax of alignments follows the **alignment API**

<http://alignapi.gforge.inria.fr>

```
alignment Alignment1 : { Class: Woman } to { Class: Person } =  
  Woman < Person  
end
```

# Alignment: Example

**ontology** S = **Class:** Person

Individual: alex Types: Person

**Class:** Child

**ontology** T = **Class:** HumanBeing

**Class:** Male **SubClassOf:** HumanBeing

**Class:** Employee

**alignment** A : S to T =

Person = HumanBeing

alex in Male

Child < not Employee

**assuming** GlobalDomain

# Networks, revisited

**network**  $N =$

$N_1, \dots, N_m, O_1, \dots, O_n, M_1, \dots, M_p, A_1, \dots, A_r$

**excluding**  $N'_1, \dots, N'_i, O'_1, \dots, O'_j, M'_1, \dots, M'_k$

- $N_i$  are other networks
- $O_i$  are OMS (possibly prefixed with labels, like  $n : O$ )
- $M_i$  are mappings (views, equivalences)
- $A_i$  are alignments

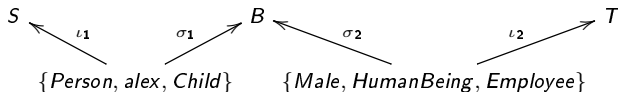
The resulting diagram  $N$  includes (institution-specific) W-alignment diagrams for each alignment  $A_i$ . Using **assuming**, assumptions about the domains of all OMS can be specified:

**SingleDomain** aligned symbols are mapped to each other

**GlobalDomain** aligned OMS a relativized

**ContextualizedDomain** alignments are reified as binary relations

# Diagram of a SingleDomain alignment



where

ontology  $B =$

Class: *Person\_HumanBeing*

Class: *Employee*

Class: *Child*

SubClassOf:  $\neg$  *Employee*

Individual: *alex*

Types: *Male*

# Resulting colimit

The colimit ontology of the diagram of the alignment above is:

**ontology B =** **Class:** *Person\_HumanBeing*

**Class:** *Employee*

**Class:** *Male* **SubClassOf:** *Person\_HumanBeing*

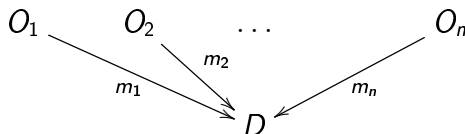
**Class:** *Child* **SubClassOf:**  $\neg$  *Employee*

**Individual:** *alex* **Types:** *Male*, *Person\_HumanBeing*

# Background: Simple semantics of diagrams

Framework: institutions like OWL, FOL, ...

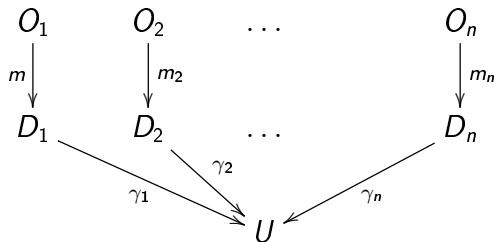
Ontologies are interpreted over the same domain



- model for  $A$ :  $(m_1, m_2)$  such that  $m_1(s) R m_2(t)$  for each  $s R t$  in  $A$
- model for a diagram: family  $(m_i)$  of models such that  $(m_i, m_j)$  is a model for  $A_{ij}$
- local models of  $O_j$  modulo a diagram:  $j$ th-projection on models of the diagram

# Integrated semantics of diagrams

Framework: different domains reconciled in a global domain



- model for a diagram: family  $(m_i)$  of models with equalizing function  $\gamma$  such that  $(\gamma_i m_i, \gamma_j m_j)$  is a model for  $A_{ij}$

# Relativization of an OWL ontology

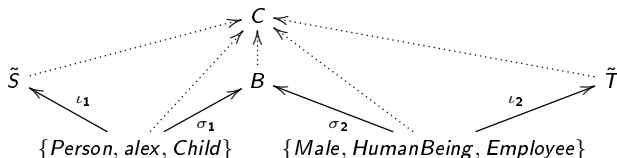
Let  $O$  be an ontology, define its relativization  $\tilde{O}$ :

- concepts are concepts of  $O$  with a new concept  $\top_O$ ;
- roles and individuals are the same
- axioms:
  - each concept  $C$  is subsumed by  $\top_O$ ,
  - each individual  $i$  is an instance of  $\top_O$ ,
  - each role  $r$  has domain and range  $\top_O$ .

and the axioms of  $O$  where the following replacement of concept is made:

- each occurrence of  $\top$  is replaced by  $\top_O$ ,
- each concept  $\neg C$  is replaced by  $\top_O \setminus C$ , and
- each concept  $\forall R.C$  is replaced by  $\top_O \sqcap \forall R.C$ .

# Example: integrated semantics



where

ontology  $B =$

Class:  $Things_S$  Class:  $Thing_T$

Class:  $Person\_HumanBeing$  SubClassOf:  $Things_S, Thing_T$

Class:  $Male$  Class:  $Employee$

Class:  $Child$  SubClassOf:  $Thing_T$  and  $\neg Employee$

Individual:  $alex$  Types:  $Male$

# Example: integrated semantics (cont'd)

ontology  $C =$

Class: *ThingS*

Class: *ThingT*

Class: *Person\_HumanBeing* **SubClassOf:** *ThingS*, *ThingC*

Class: *Male* **SubClassOf:** *Person\_HumanBeing*

Class: *Employee* **SubClassOf:** *ThingT*

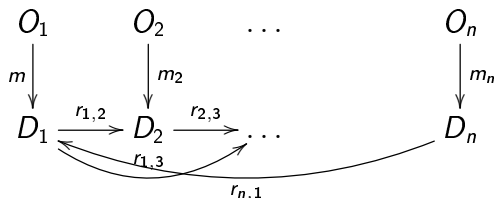
Class: *Child* **SubClassOf:** *ThingS*

Class: *Child* **SubClassOf:** *ThingT* **and**  $\neg$  *Employee*

Individual: *alex* **Types:** *Male*, *Person\_HumanBeing*

# Contextualized semantics of diagrams

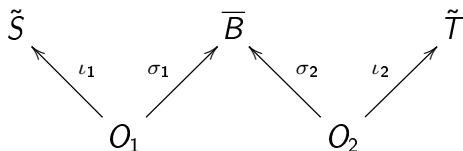
Framework: different domains related by coherent relations



such that

- $r_{ij}$  is functional and injective,
- $r_{ii}$  is the identity (diagonal) relation,
- $r_{ji}$  is the converse of  $r_{ij}$ , and
- $r_{ik}$  is the relational composition of  $r_{ij}$  and  $r_{jk}$
- model for a diagram: family  $(m_i)$  of models with coherent relations  $(r_{ij})$  such that  $(m_i, r_{ji}m_j)$  is a model for  $A_{ij}$

# Contextualized semantics of diagrams, revisited



where  $\overline{B}$  modifies  $B$  as follows:

- $r_{ij}$  are added to  $\overline{B}$  as roles with domain  $\top_S$  and range  $\top_T$
- the correspondences are translated to axioms involving these roles:
  - $s_i = t_j$  becomes  $s_i \ r_{ij} \ t_j$
  - $a_i \in c_j$  becomes  $a_i \in \exists r_{ij}.c_j$
  - ...
- the properties of the roles are added as axioms in  $\overline{B}$

# Adding domain relations to the bridge

ontology  $\overline{B}$  =

Class: *ThingS*

Class: *ThingT*

ObjectProperty:  $r_{ST}$  Domain: *ThingS* Range: *ThingT*

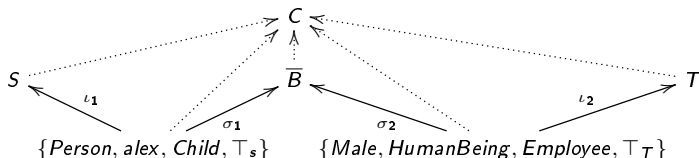
Class: *Person* EquivalentTo:  $r_{ST}$  some *HumanBeing*

Class: *Employee*

Class: *Child* SubClassOf:  $r_{ST}$  some  $\neg$  *Employee*

Individual: *alex* Types:  $r_{ST}$  some *Male*

# Example: contextualized semantics



where

ontology  $\mathbb{C} =$

Class: *ThingS*

Class: *ThingT*

ObjectProperty:  $r_{ST}$  Domain: *ThingS* Range: *ThingT*

Class: *Person* EquivalentTo:  $r_{ST}$  some *HumanBeing*

Class: *Employee*

Class: *Child* SubClassOf:  $r_{ST}$  some  $\neg$  *Employee*

Individual: *alex* Types:  $r_{ST}$  some *Male*, *Person*

# Syntax of equivalences

```
EquivalenceDefinition ::= 'equivalence' EquivalenceName ':'  
                        EquivalenceType 'end'  
EquivalenceName      ::= IRI  
EquivalenceType      ::= GroupOMS '<->' GroupOMS '=' OMS  
                        | Network '<->' Network '=' Network
```

# Equivalences

- **equivalence**  $Id : O_1 \leftrightarrow O_2 = O_3$
- (fragment) OMS  $O_3$  is such that  $O_i$  then %def  $O_3$  is a definitional extension of  $O_i$  for  $i = 1, 2$ ;
- this implies that  $O_1$  and  $O_2$  have model classes that are in bijective correspondence

**equivalence** e : algebra:BooleanAlgebra  
 $\leftrightarrow$  algebra:BooleanRing =

$$x \wedge y = x \cdot y$$

$$x \vee y = x + y + x \cdot y$$

$$\neg x = 1 + x$$

$$x \cdot y = x \wedge y$$

$$x + y = (x \vee y) \wedge \neg(x \wedge y)$$

**end**

# Syntax of module relations

```
ModuleRelDefinition ::= 'module' ModuleName [Conservative] ':'  
                      ModuleType 'for' InterfaceSignature  
ModuleName          ::= IRI  
ModuleType           ::= GroupOMS 'of' GroupOMS
```

# Module Relations

- **module** *Id*  $c : O_1$  **of**  $O_2$  **for**  $\Sigma$
- $O_1$  is a module of  $O_2$  with restriction signature  $\Sigma$  and conservativity  $c$ 
  - $c = \%mcons$  every  $\Sigma$ -reduct of an  $O_1$ -model can be expanded to an  $O_2$ -model
  - $c = \%ccons$  every  $\Sigma$ -sentence  $\varphi$  following from  $O_1$  already follows from  $O_2$

This relation shall hold for any module  $O_1$  extracted from  $O_2$  using the **extract** construct.

# Syntax of queries (only informative annex!)

```

Term                ::= ($<$)an expression specific to an OMS language($>$)
GeneralizedTerm      ::= Term | SymbolRef
QueryRelatedDefinition ::= QueryDefinition
                        | SubstitutionDefinition
                        | ResultDefinition
QueryDefinition      ::= 'query' QueryName '=' 'select' Vars 'where'
                        Sentence 'in' GroupOMS
                        ['along' OMSLanguageTranslation] 'end'
SubstitutionDefinition ::= 'substitution' SubstitutionName ':'
                        GroupOMS 'to' GroupOMS '=' SymbolMap
                        'end'
ResultDefinition     ::= 'result' ResultName '=' SubstitutionName
                        ( ',' SubstitutionName )* 'for' QueryName
                        ['%complete'] 'end'
OMS                  ::= ($...$) | OMS 'with' SubstitutionName
QueryName            ::= IRI
SubstitutionName     ::= IRI
ResultName           ::= IRI
Vars                 ::= Symbol ( ',' Symbol )*
```

# Queries

DOL is a logical (meta) language

- focus on ontologies, models, specifications,
- and their logical relations: logical consequence, interpretations,  
...

Queries are different:

- answer is not “yes” or “no”, but an answer substitution
- query language may differ from language of OMS that is queried

# Sample query languages

- conjunctive queries in OWL
- Prolog/Logic Programming
- SPARQL

# Syntax of queries in DOL

New OMS declarations and relations:

```
query qname = select vars where sentence in OMS
                [along language-translation]
substitution sname : OMS1 to OMS2 = derived-symbol-map
result rname = sname_1, ..., sname_n for qname
                %% result is a substitution
```

New sentences (however, as structured OMS!):

```
apply(sname,sentence)    %% apply substitution
```

Open question: how to deal with “construct” queries?

# Proof calculus

# Structured specifications over an arbitrary institution (covers part of DOL OMS)

$SP$	$::=$	$\langle \Sigma, \Gamma \rangle$	basic specification
	$ $	$SP \cup SP$	union
	$ $	$\sigma(SP)$	translation
	$ $	$SP _{\sigma}$	hiding

## ... and their semantics

## Definition (Signature and model class of a specification)

$$\text{Sig}(\langle \Sigma, \Gamma \rangle) = \Sigma$$

$$\text{Mod}(\langle \Sigma, \Gamma \rangle) = \{M \in \text{Mod}(\Sigma) \mid M \models \Gamma\}$$

$$\text{Sig}(SP_1 \cup SP_2) = \text{Sig}(SP_1) = \text{Sig}(SP_2)$$

$$\text{Mod}(SP_1 \cup SP_2) = \text{Mod}(SP_1) \cap \text{Mod}(SP_2)$$

$$\text{Sig}(\sigma: \Sigma_1 \longrightarrow \Sigma_2(SP)) = \Sigma_2$$

$$\text{Mod}(\sigma(SP)) = \{M \in \text{Mod}(\Sigma_2) \mid M|_{\sigma} \in \text{Mod}(SP)\}$$

$$\text{Sig}(SP|_{\sigma: \Sigma_1 \longrightarrow \Sigma_2}) = \Sigma_1$$

$$\text{Mod}(SP|_{\sigma: \Sigma_1 \longrightarrow \Sigma_2}) = \{M|_{\sigma} \mid M \in \text{Mod}(SP)\}$$

## Definition (Logical consequence, specification refinement)

$$SP \models \varphi \quad \text{iff} \quad M \models \varphi \text{ for all } M \in \text{Mod}(SP)$$

# Entailment systems

## Definition

Given an institution  $\mathcal{I} = (\mathbf{Sign}, \mathbf{Sen}, Mod, \models)$ , an *entailment system*  $\vdash$  for  $\mathcal{I}$  consists of relations  $\vdash_{\Sigma} \subseteq \mathcal{P}(\mathbf{Sen}(\Sigma)) \times \mathbf{Sen}(\Sigma)$  such that

- ① *reflexivity*: for any  $\varphi \in \mathbf{Sen}(\Sigma)$ ,  $\{\varphi\} \vdash_{\Sigma} \varphi$ ,
- ② *monotonicity*: if  $\Gamma \vdash_{\Sigma} \varphi$  and  $\Gamma' \supseteq \Gamma$  then  $\Gamma' \vdash_{\Sigma} \varphi$ ,
- ③ *transitivity*: if  $\Gamma \vdash_{\Sigma} \varphi_i$  for  $i \in I$  and  $\Gamma \cup \{\varphi_i \mid i \in I\} \vdash_{\Sigma} \psi$ , then  $\Gamma \vdash_{\Sigma} \psi$ ,
- ④  *$\vdash$ -translation*: if  $\Gamma \vdash_{\Sigma} \varphi$ , then for any  $\sigma: \Sigma \longrightarrow \Sigma'$  in  $\mathbf{Sign}$ ,  $\sigma(\Gamma) \vdash_{\Sigma'} \sigma(\varphi)$ ,
- ⑤ *soundness*: if  $\Gamma \vdash_{\Sigma} \varphi$  then  $\Gamma \models_{\Sigma} \varphi$ .

The entailment system is *complete* if, in addition,  
 $\Gamma \models_{\Sigma} \varphi$  implies  $\Gamma \vdash_{\Sigma} \varphi$ .

# Proof calculus for entailment (Borzyszkowski)

$$(CR) \frac{\{SP \vdash \varphi_i\}_{i \in I} \quad \{\varphi_i\}_{i \in I} \vdash \varphi}{SP \vdash \varphi}$$

$$(basic) \frac{\varphi \in \Gamma}{\langle \Sigma, \Gamma \rangle \vdash \varphi}$$

$$(sum1) \frac{SP_1 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi}$$

$$(sum2) \frac{SP_1 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi}$$

$$(trans) \frac{SP \vdash \varphi}{\sigma(SP) \vdash \sigma(\varphi)}$$

$$(derive) \frac{SP \vdash \sigma(\varphi)}{SP|_{\sigma} \vdash \varphi}$$

Soundness means:  $SP \vdash \varphi$  implies  $SP \models \varphi$

Completeness means:  $SP \models \varphi$  implies  $SP \vdash \varphi$

# Proof calculus for refinement (Borzyszkowski)

$$(Basic) \frac{SP \vdash \Gamma}{\langle \Sigma, \Gamma \rangle \rightsquigarrow SP}$$

$$(Sum) \frac{SP_1 \rightsquigarrow SP \quad SP_2 \rightsquigarrow SP}{SP_1 \cup SP_2 \rightsquigarrow SP}$$

$$(Trans) \frac{SP \rightsquigarrow SP'|_{\sigma}}{\sigma(SP) \rightsquigarrow SP'}$$

$$(Derive) \frac{SP \rightsquigarrow SP''}{SP|_{\sigma} \rightsquigarrow SP'} \quad \begin{array}{l} \text{if } \sigma: SP' \longrightarrow SP'' \\ \text{is a conservative extension} \end{array}$$

Soundness means:  $SP_1 \rightsquigarrow SP_2$  implies  $SP_1 \rightsquigarrow\rightsquigarrow SP_2$

Completeness means:  $SP_1 \rightsquigarrow\rightsquigarrow SP_2$  implies  $SP_1 \rightsquigarrow SP_2$

# Craig-Robinson interpolation

## Definition

 $\Psi$ 

$$\begin{array}{ccc} \Sigma & \xrightarrow{\varphi_1} & \Sigma_1 \\ \varphi_2 \downarrow & & \downarrow \theta_1 \\ \Sigma_2 & \xrightarrow{\theta_2} & \Sigma' \end{array}$$

$$(1) \Psi_1 \models \varphi_1(\Psi)$$

$$(2) \varphi_2(\Psi) \cup \Gamma_2 \models \Psi_2 \quad (0) \theta_1(\Psi_1) \cup \theta_2(\Gamma_2) \models \theta_2(\Psi_2)$$

A commutative square admits *Craig-Robinson interpolation*,  
 if for all finite  $\Psi_1 \subseteq \text{Sen}(\Sigma_1)$ ,  $\Psi_2, \Gamma_2 \subseteq \text{Sen}(\Sigma_2)$ ,  
 if (0), then there exists a finite  $\Psi \subseteq \text{Sen}(\Sigma)$  with (1) and (2).

$\mathcal{I}$  has *Craig-Robinson interpolation* if all signature pushouts admit  
 Craig-Robinson interpolation.

# Soundness and Completeness

## Theorem (Borzyszkowski, Tarlecki, Diaconescu)

*Under the assumptions that*

- *the institution admits **Craig-Robinson interpolation**,*
- *the institution is **weakly semi-exact**, and*
- *the entailment system is **complete**,*

*the calculus for structured entailment and refinement is sound and complete.*

For refinement, we need an **oracle for conservative extensions**.

Weak semi-exactness = *Mod* maps pushouts to weak pullbacks

**Problem: Craig interpolation often fails:**

- many-sorted FOL (with non-injective signature morphisms)
- many-sorted equational logic

# Structured normal form

$$\overline{snf(\langle \Sigma, \Gamma \rangle)} = \langle \Sigma, \Gamma \rangle|_{id}$$

$$\frac{snf(SP_1) = SP'_1|_{\sigma_1} \quad snf(SP_2) = SP'_2|_{\sigma_2}}{snf(SP_1 \cup SP_2) = (\theta_1(SP'_1) \cup \theta_2(SP'_2))|_{\sigma_1; \theta_1}} \quad \text{if} \quad \begin{array}{ccc} Sig[SP_1] & \xrightarrow{\sigma_1} & Sig[SP'_1] \\ \downarrow \sigma_2 & & \downarrow \theta_1 \\ Sig[SP'_2] & \xrightarrow{\theta_2} & \Sigma' \end{array}$$

$$\frac{snf(SP) = SP'|_{\sigma_1}}{snf(\sigma_2(SP)) = (\theta_1(SP'))|_{\theta_2}} \quad \text{if} \quad \begin{array}{ccc} Sig[SP] & \xrightarrow{\sigma_1} & Sig[SP'] \\ \downarrow \sigma_2 & & \downarrow \theta_1 \\ \Sigma_2 & \xrightarrow{\theta_2} & \Sigma' \end{array} \quad \text{is a pushout}$$

$$\frac{snf(SP) = SP'|_{\sigma}}{snf(SP|_{\theta}) = SP'|_{\theta; \sigma}}$$

# Properties of the structured normal form

## Proposition

*In any weakly semi-exact institution,  $SP$  and  $\text{snf}(SP)$  are equivalent.*

Moreover, we can obtain a stronger completeness result:

## Theorem

*Under the assumptions that the institution is weakly semi-exact and the entailment system is complete, the calculi for specification entailments and refinement between structured specifications extended by the following structured normal form rule:*

$$(\text{snf}) \frac{SP' \vdash \sigma(\varphi)}{SP \vdash \varphi} \quad \text{if } \text{snf}(SP) = SP'|_{\sigma}$$

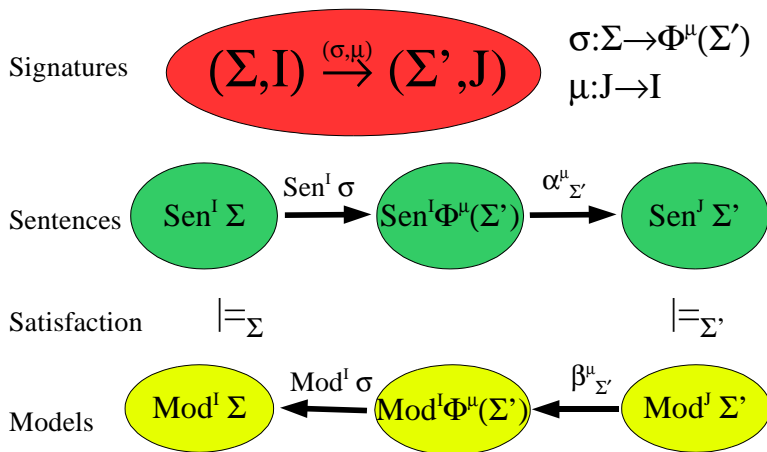
# Heterogeneous specification

## Definition

A *heterogeneous logical environment* ( $\mathcal{HLE}$ ) (or indexed coinstitution) is diagram of institutions and comorphisms.

# Grothendieck institution over an $\mathcal{HLE}$

## The Grothendieck Institution



# Heterogeneous structuring operations

*heterogeneous translation*: For any  $\mathcal{I}$ -specification  $SP$ ,  $\rho(SP)$  is a specification with:

$$\text{Sig}[\rho(SP)] := \Phi(\text{Sig}[SP])$$

$$\text{Mod}[\rho(SP)] := \beta_{\text{Sig}[SP]}^{-1}(\text{Mod}[SP])$$

*heterogeneous hiding*: For any  $\mathcal{I}'$ -specification  $SP'$  and signature  $\Sigma$  with  $\text{Sig}[SP'] = \Phi(\Sigma)$ ,  $SP'|_{\rho}^{\Sigma}$  is a specification with:

$$\text{Sig}[SP'|_{\rho}^{\Sigma}] := \Sigma$$

$$\text{Mod}[SP'|_{\rho}^{\Sigma}] := \beta_{\Sigma}(\text{Mod}[SP'])$$

This can be interpreted as structuring in the Grothendieck institution.

# A heterogeneous proof calculus

$$(het-trans) \frac{SP \vdash \varphi}{\rho(SP) \vdash \alpha(\varphi)}$$

$$(borrowing) \frac{\rho(SP) \vdash \alpha(\varphi)}{SP \vdash \varphi}$$

$$(Het-snf) \frac{SP' \vdash \sigma(\alpha(\varphi))}{SP \vdash \varphi}$$

$$(het-derive) \frac{SP \vdash \alpha(\varphi)}{SP|_{\rho}^{\Sigma} \vdash \varphi}$$

if  $\rho$  is model-expansive

if  $hsnf(SP) = (SP'|_{\sigma})|_{\rho}^{\Sigma}$

# A heterogeneous proof calculus for refinement

$$\begin{array}{l}
 (\textit{Het-Trans}) \quad \frac{SP \rightsquigarrow SP' |_{\rho}^{\Sigma}}{\rho(SP) \rightsquigarrow SP'} \\
 (\textit{Het-Derive}) \quad \frac{SP \rightsquigarrow SP''}{SP |_{\rho}^{\Sigma} \rightsquigarrow SP'} \quad \text{if } \rho: SP' \longrightarrow SP'' \text{ is a conservative extension}
 \end{array}$$

Conservativity of  $\rho = (\Phi, \alpha, \beta): SP' \longrightarrow SP''$  means that for each model  $M' \in \textit{Mod}(SP')$ , there is a model  $M'' \in \textit{Mod}(SP'')$  with  $\beta(M'') = M'$ .

# Heterogeneous completeness

## Theorem

*For a lax heterogeneous logical environment  $\mathcal{HLE}: \mathcal{G} \longrightarrow \text{coINS}$  (with some of the institutions having entailment systems), the proof calculi for heterogeneous specifications are sound for  $\mathcal{I}^{\mathcal{HLE}}/\equiv$ . If*

- ①  *$\mathcal{HLE}$  is lax-quasi-exact,*
- ② *all institution comorphisms in  $\mathcal{HLE}$  are weakly exact,*
- ③ *there is a set  $\mathcal{L}$  of institutions in  $\mathcal{HLE}$  that come with complete entailment systems,*
- ④ *all institutions in  $\mathcal{L}$  are quasi-semi-exact,*
- ⑤ *from each institution in  $\mathcal{HLE}$ , there is some model-expansive comorphism in  $\mathcal{HLE}$  going into some institution in  $\mathcal{L}$ ,*

*the proof calculus for entailments between heterogeneous specifications and sentences is complete over  $\mathcal{I}^{\mathcal{HLE}}/\equiv$ . If, moreover,*

# Tool support

# Tool support: Heterogeneous Tool Set (Hets)

- available at <http://hets.eu>
- speaks DOL, HetCASL, CoCASL, CspCASL, MOF, QVT, OWL, Common Logic, and other languages
- analysis
- computation of colimits
- management of proof obligations
- interfaces to theorem provers, model checkers, model finders

# Tool support: Ontohub web portal and repository

**Ontohub** is a web-based repository engine for distributed heterogeneous (multi-language) OMS

- prototype available at `ontohub.org`
- speaks DOL, OWL, Common Logic, and other languages
- mid-term goal: follow the Open Ontology Repository Initiative (OOR) architecture and API
- API is discussed at `https://github.com/ontohub/OOR\_Ontohub\_API`
- annual Ontology summit as a venue for review, and discussion

# Conclusion

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- DOL is a **meta language** for (formal) ontologies, specifications and models (**OMS**)
- DOL covers many aspects of modularity of and relations among OMS ("**OMS-in-the large**")
- DOL will be submitted to the OMG as an answer to the **OntoOp** RFP
- **you** can help with joining the **OntoOp** discussion
  - see [ontoiop.org](http://ontoiop.org)

# Challenges

- What is a suitable abstract meta framework for **non-monotonic** logics and **rule languages** like RIF and RuleML? Are institutions suitable here? different from those for OWL?
- What is a useful abstract notion of **query** (language) and **answer substitution**?
- How to integrate TBox-like and ABox-like OMS?
- Can the notions of **class hierarchy** and of **satisfiability** of a class be **generalised** from OWL to other languages?
- How to interpret alignment correspondences with confidence other than 1 in a combination?
- Can **logical frameworks** be used for the specification of OMS languages and translations?
- **Proof support for whole of DOL**

# Related work

- Structured specifications and their semantics (Clear, ASL, CASL, ...)
- Heterogeneous specification (HetCASL)
- modular ontologies (WoMo workshop series)